THE UNIVERSITY of TEXAS
HEALTH SCIENCE CENTER AT HOUSTON
SCHOOL of HEALTH INFORMATION SCIENCES

Image Display and Histograms

For students of HI 5323
“Image Processing”

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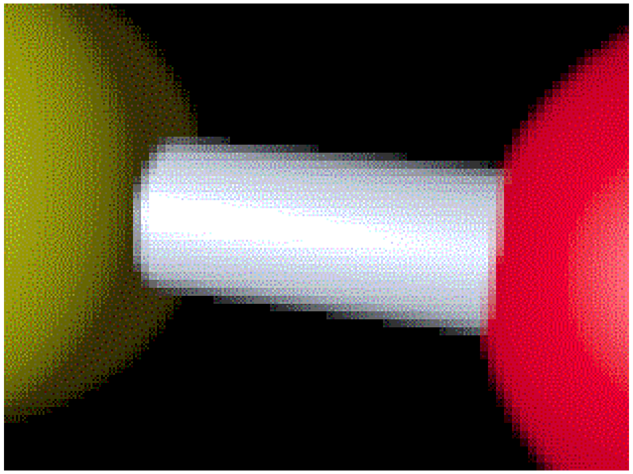
http://biomachina.org/courses/processing/02.html

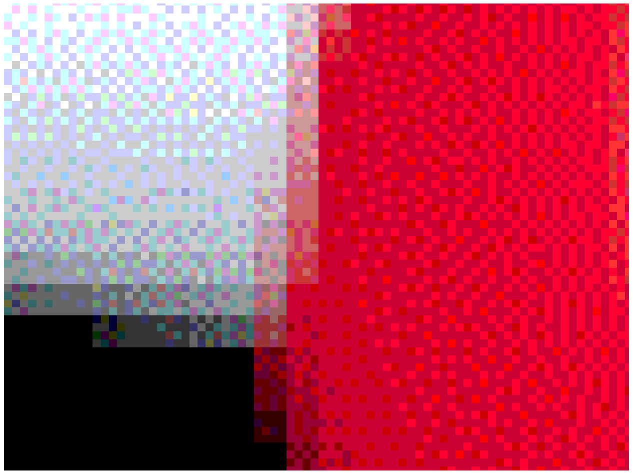
Properties of Displays

- Size and # of Pixels
- Brightness
- Linearity
- Flatness
- Resolution

Volatile Display vs. Permanent Display

- Volatile display
  - Display continually refreshed from a stored digital image
- Permanent display
  - Color printing
  - Dithering: Image colors that were defined in the higher definition color space, but that are not available in the lower definition color space, are approximated by a dot pattern which arranges different colors from the lower definition palette in a pixel array to create a perceptual approximation of the unavailable color.





Intensity Discrimination

- Human eye can discriminate 1000 shades of gray
- For constant adaptation, about 200 levels
- 8 bits → $2^8 = 256$ shades

Linearity

- Applies to output as well as input
- Twice the recorded value should be twice as bright
- Problem: monitor response not linear

$$I = cV^{\gamma} + b$$

Gamma

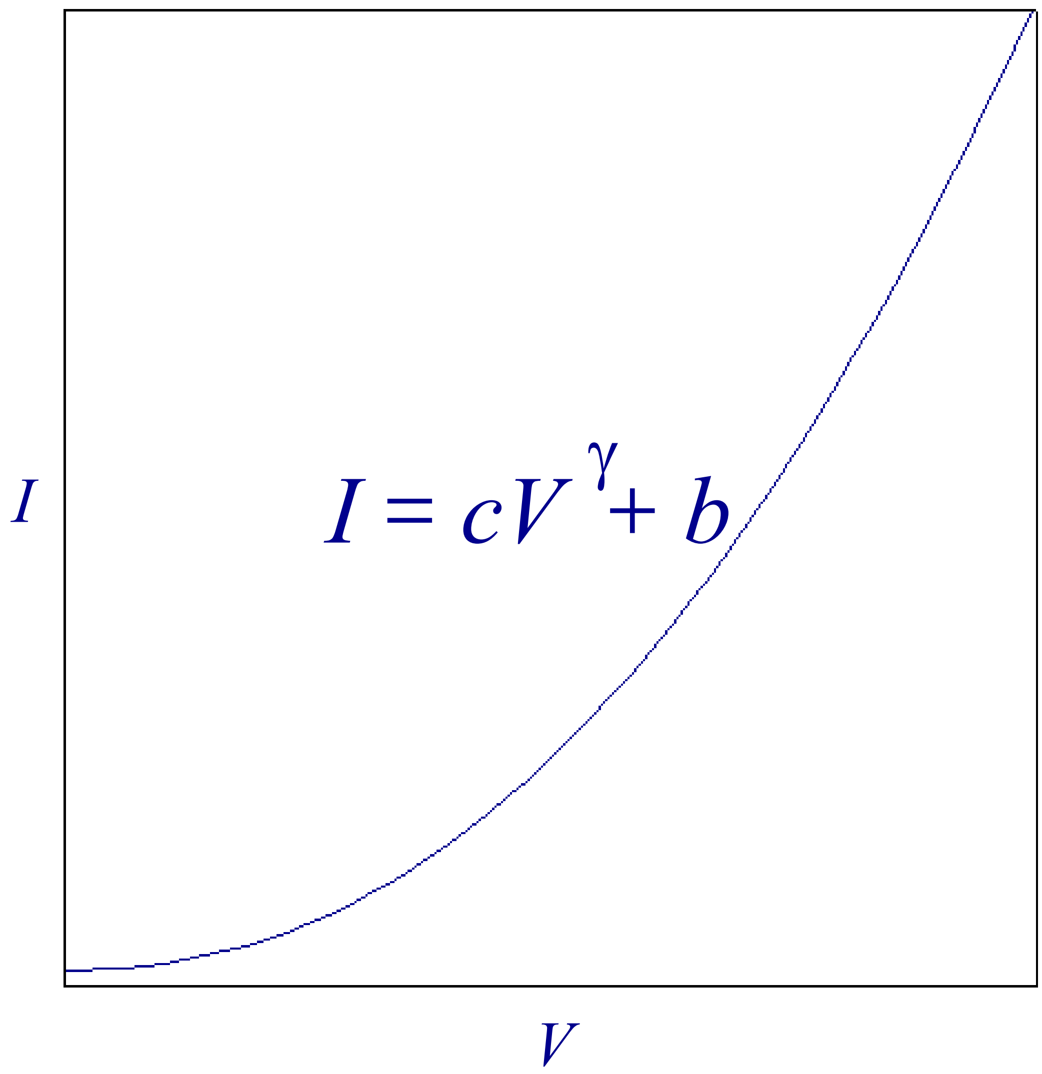
Output Intensity

Offset (bias)

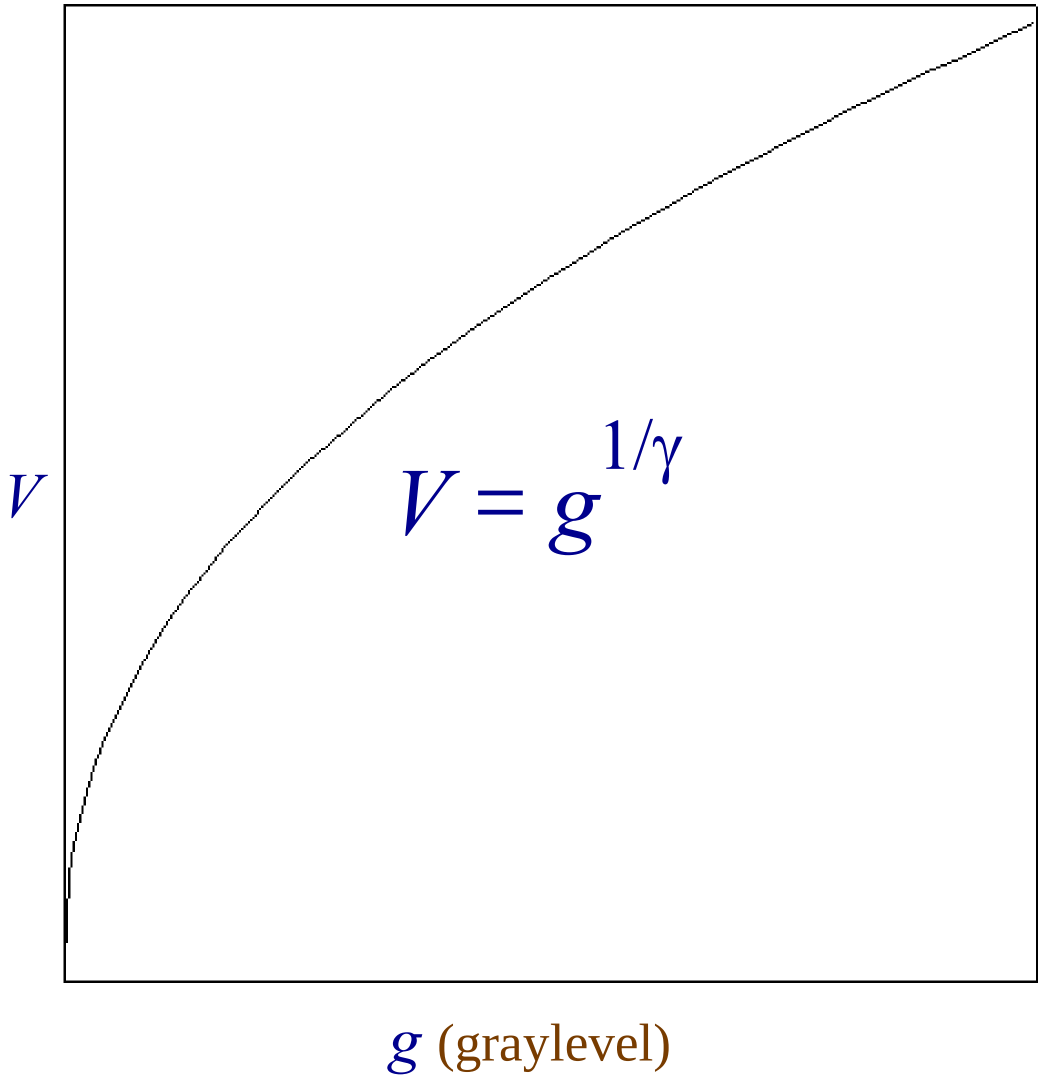
Gain (slope)

Input Voltage

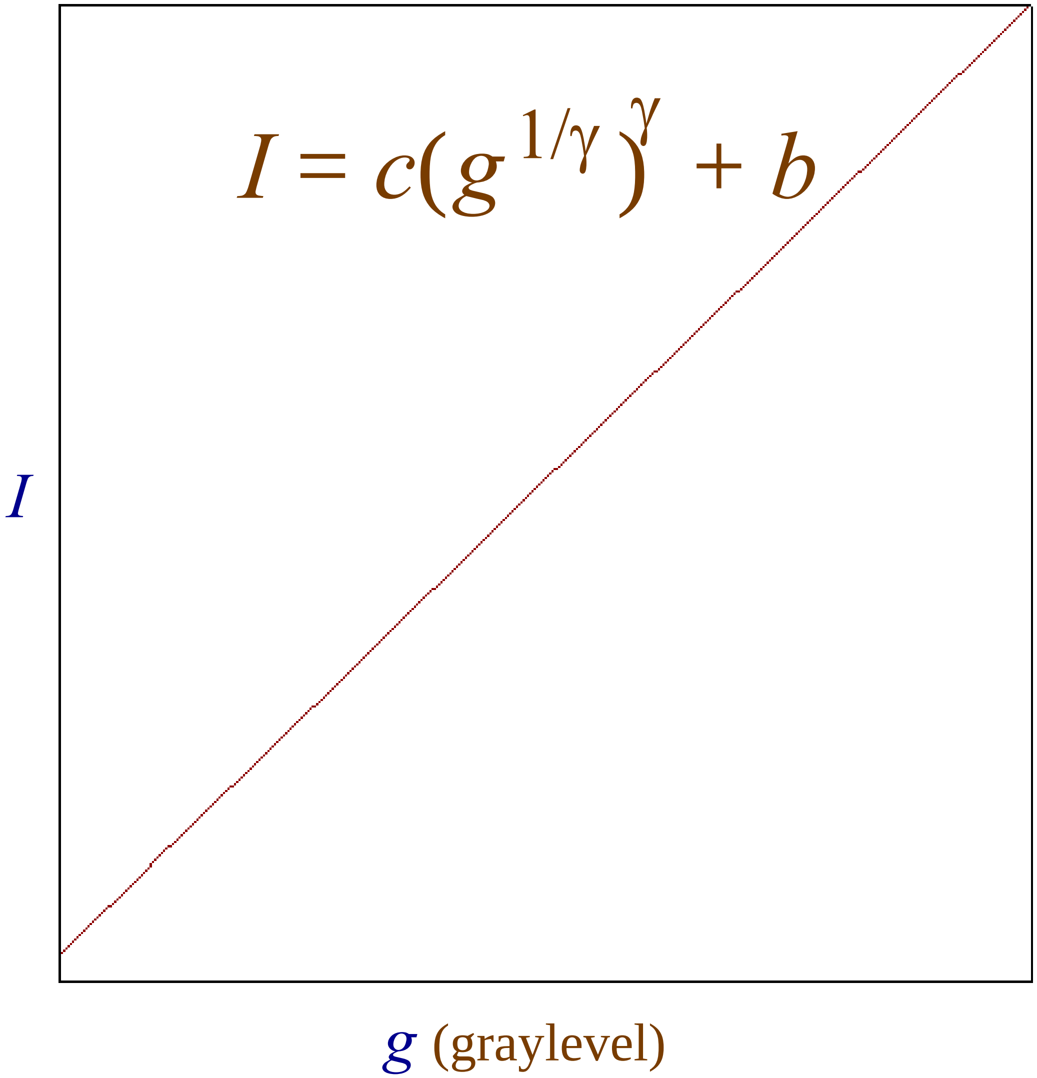
Gamma Response



Gamma Correction



After Gamma Correction



Gamma Correction

- Different monitors have different gammas
- Be careful of different operating systems, drivers, etc.
- Applies to imaging devices as well: cameras, scanners, etc.

Gaussian Spots

- Digital display devices generate output via a collection of dots/spots
- Each spot has a 2-D intensity distribution:
  - Modeled as a radially symmetric Gaussian

$$p(x, y) = e^{-(x^2 + y^2)} = e^{-r^2}$$

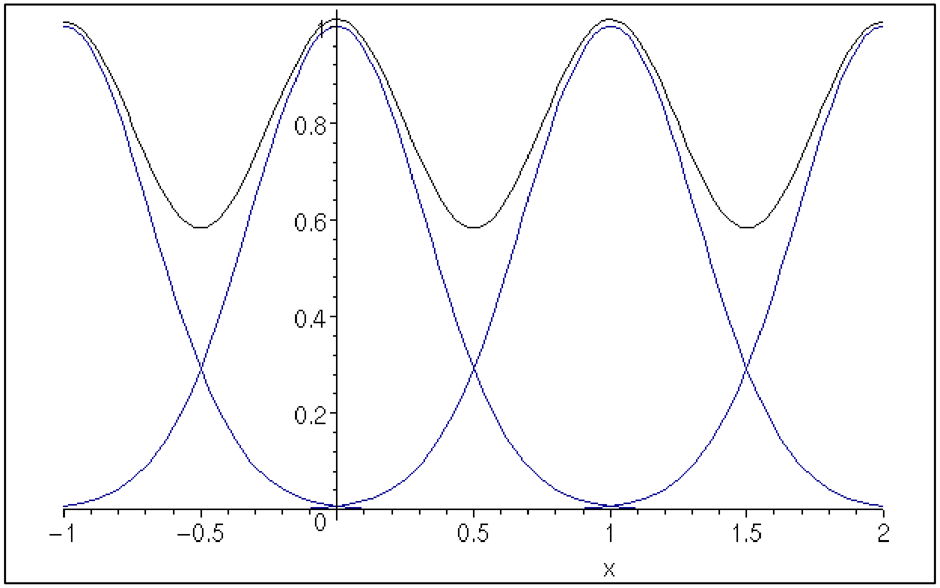
- *R* → radius at which intensity drops to ½ maximum

$$p(r) = e^{-r^2} = e^{-(r/R)^2 \ln(2)}$$

$$= e^{\ln(2^{-(r/R)^2})} = 2^{-(r/R)^2}$$

Flat Display

- A constant (high-intensity) image should look flat
- Problem: Hard to make individual spots blend into a constant field



Flat Display

- A constant (high-intensity) image should look flat
- Problem: Hard to make individual spots blend into a constant field

- Solution
  - Use wide spots
  - Put spots close together

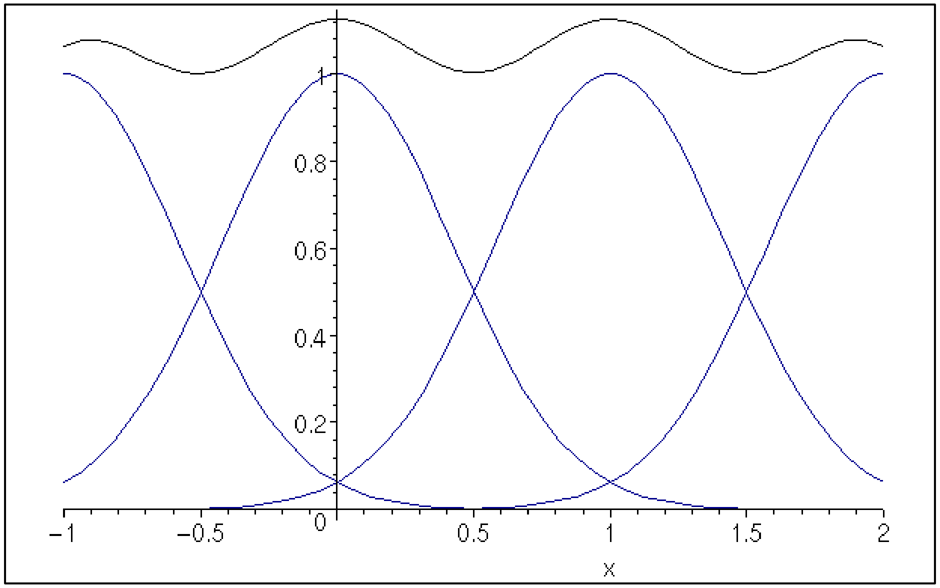
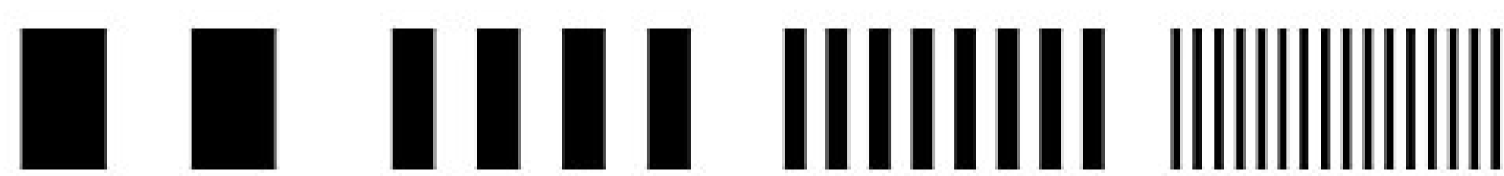


Image Resolution and Contrast

- Wider (or closer together) spots mean less resolution/sharpness
- Individual spots spread and interact with neighbor
- Rapid changes lose contrast



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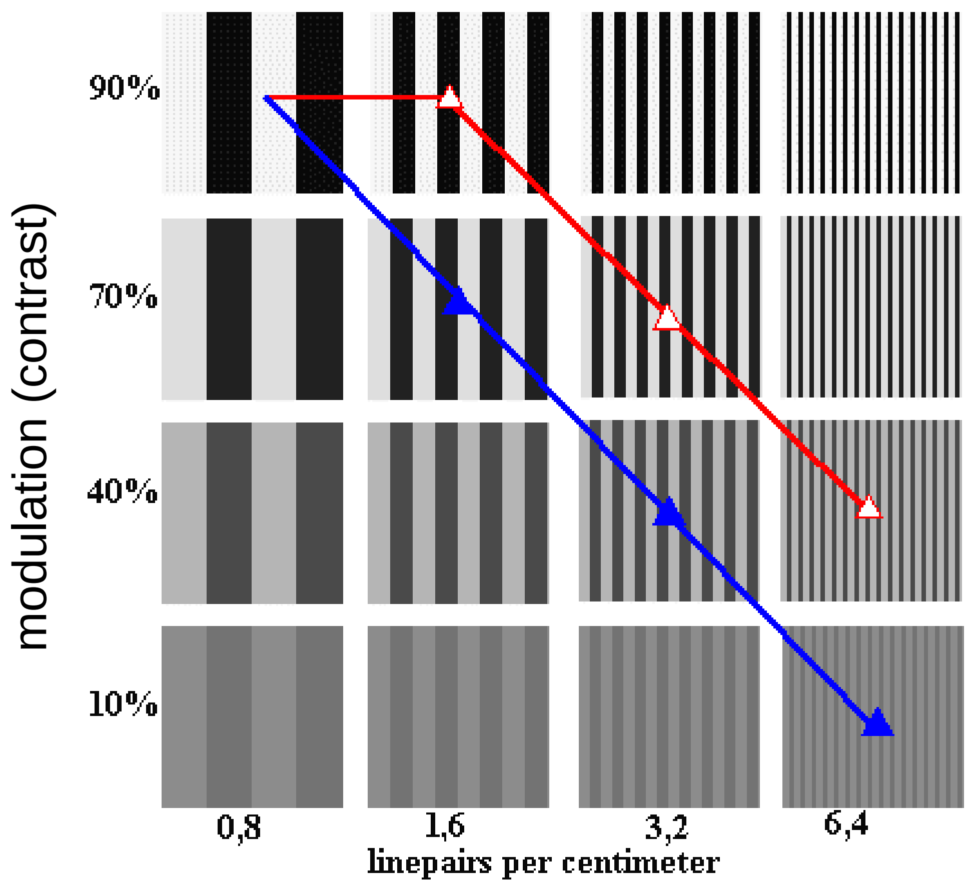
- Modulation: scaled contrast between neighboring high and low intensity pixels

Contrast vs. Frequency

Two test displays



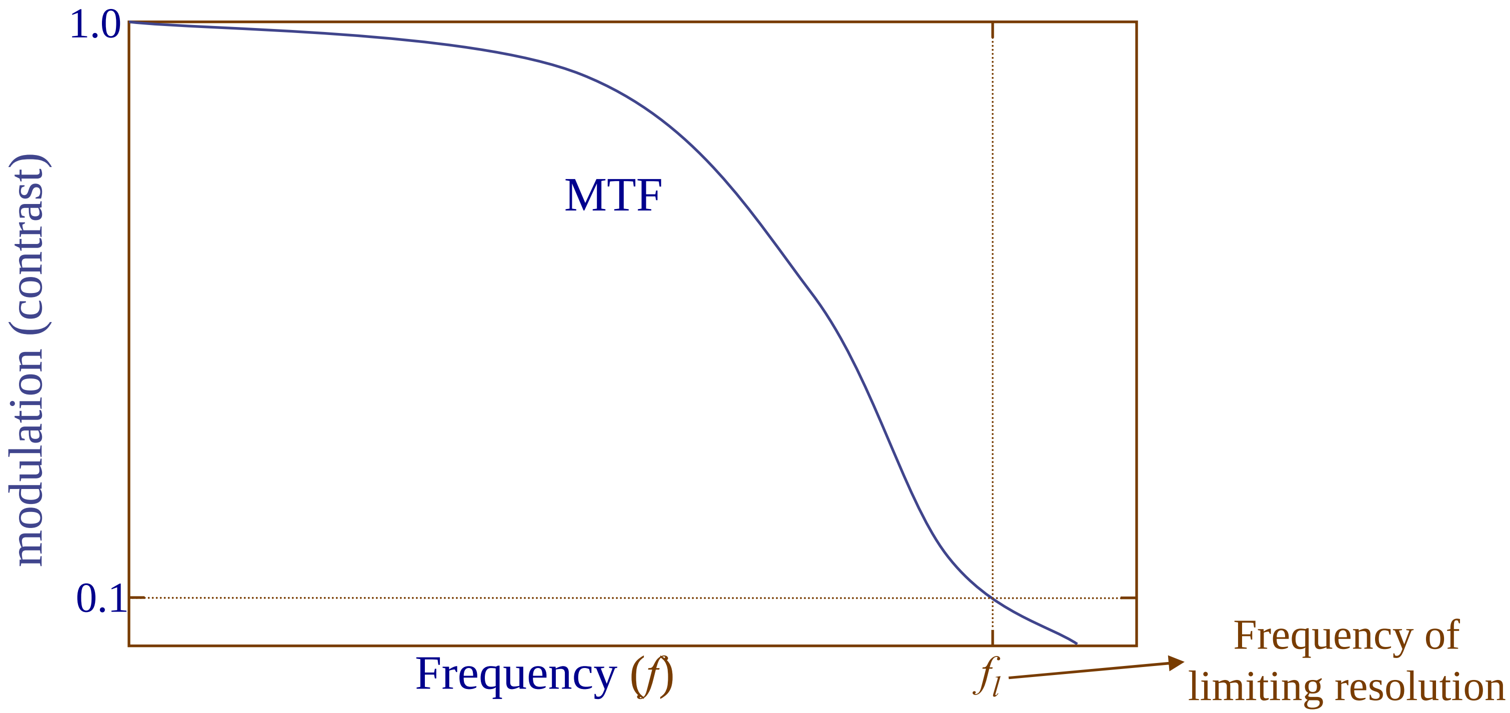
fineness of definition

- Limiting fineness depends on contrast
- Display quality is defined by 2D curve: modulation vs. fineness (frequency)

Modulation Contrast Function

- Instead of line pairs, use sine waves
- Measure contrast (modulation) as a function of spatial frequency of sine wave

MTF of a Lens or Display



Eric Mortenson 2001

In an imaging system one would like to achieve the highest possible contrast with the greatest possible fineness of definition, distributed as evenly as possible over the entire image field.

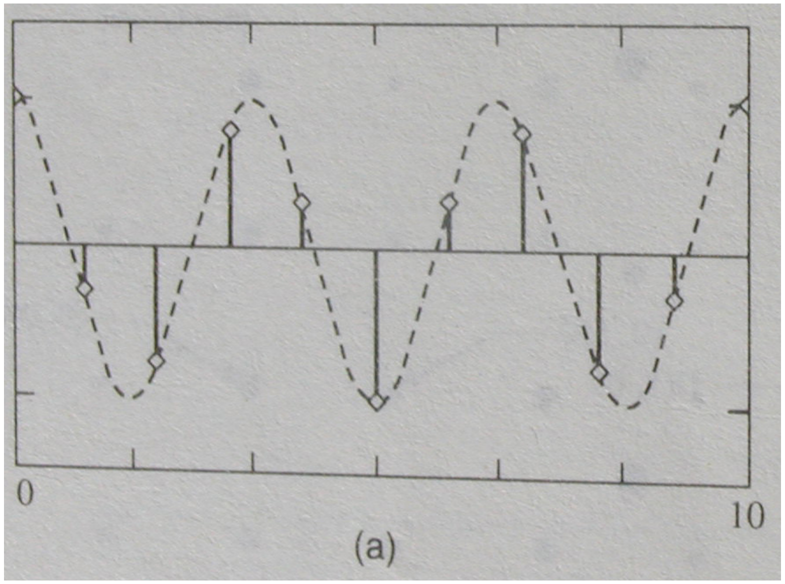
Noise

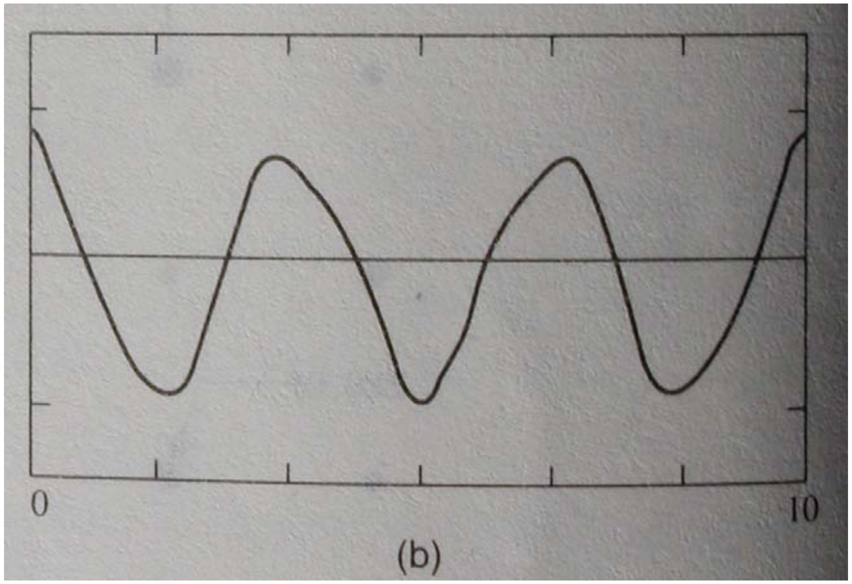
- Intensity of display spot
  - Random noise
  - Periodic and synchronized noise
- Position of display spot
  - Effects of spot interaction + position noise

Reconstruction

- Reverse of digitization:
- Undo sampling: or at least make is seem continuous
  - Gaussian spots
  - Resampling
- Undo quantization: convert back to analog
  - Interpolation
  - Dithering

Interpolation





(a) The cosine sample at 3.3 sample per cycle

(b) The sampled cosine interpolated with a Gaussian display spot

Oversampling & Resampling

- The inappropriate shape of the Gaussian display spot has less effect when there are more sample points per cycle of the cosine
  - Oversampling

    tradeoff – more expensive

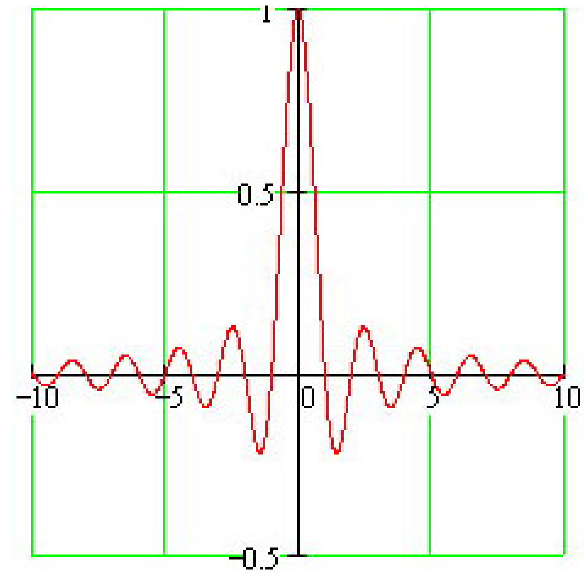
  - Resampling
    - The process of increasing the size of the image by digitally implemented interpolation prior to displaying it
    - A 512 x 512 image might be interpolated up to 1024 x 1024, then displayed on a monitor with a Gaussian display spot

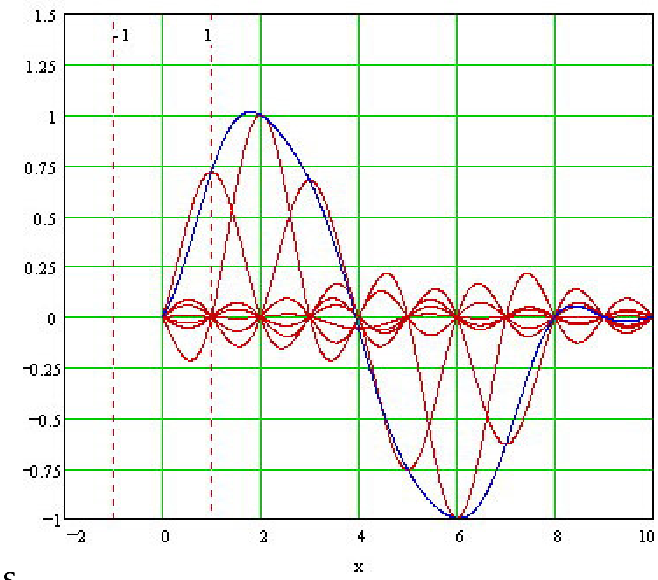
Sinc Interpolation

- Interpolation function has form

  sinc (x) = sin(πx)/πx

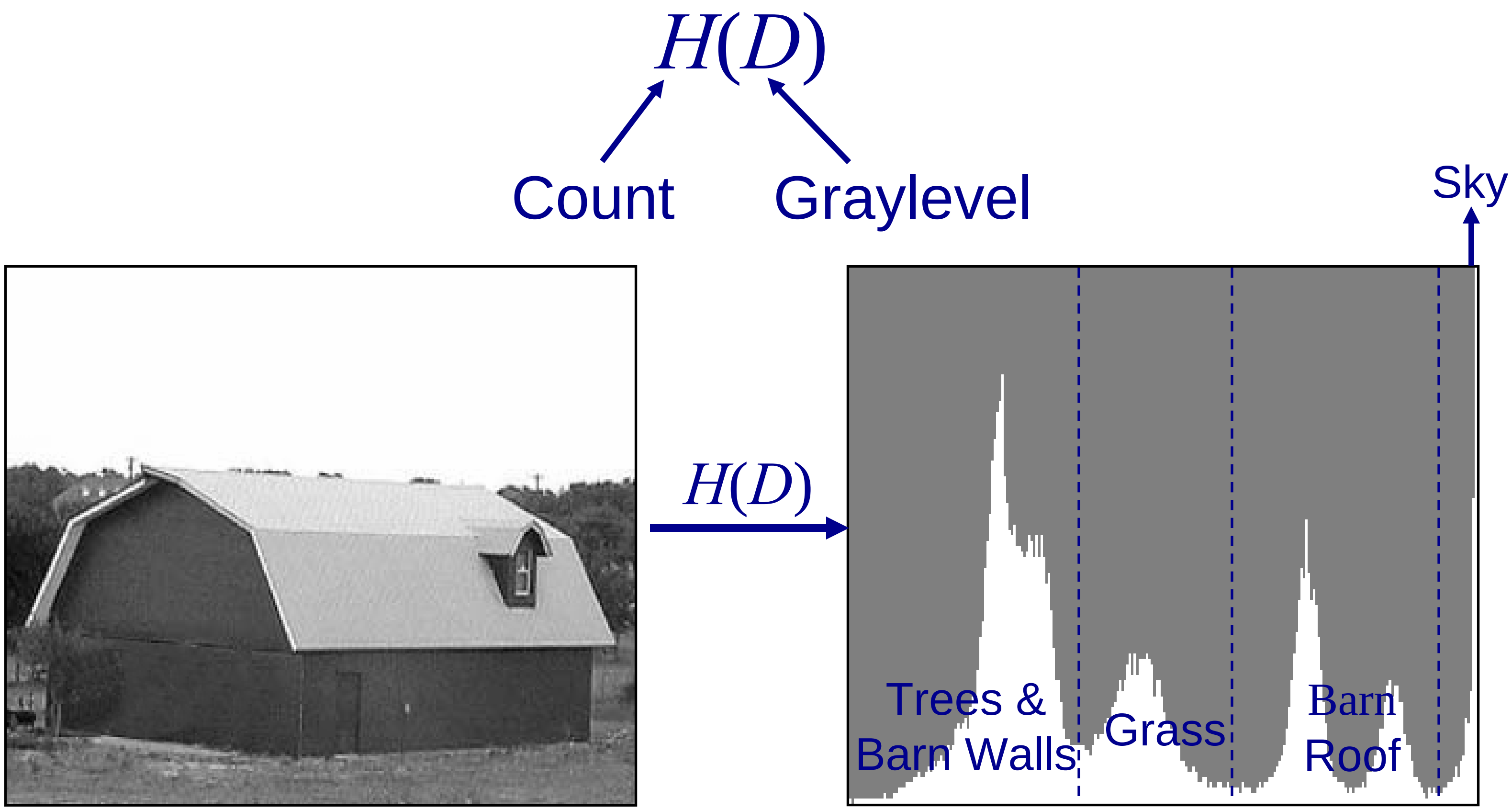
  Sinc interpolation is designed to minimize aliasing in a signal. It turns out that sinc(0)=1 but its value at any other integer is zero. Each sample's contribution to the signal is a sinc function centered on the sample. The sinc function is scaled to match the height of the sample. The frequency of the sinc function is set to match the sample rate so that all neighboring samples occur where the sinc function goes to zero, at integer values. The overall signal is the sum of all of the sinc functions of all of the samples.





Histograms

- Histograms count the number of occurrences of each graylevel value



Properties

- Sum of histogram elements equals the image size:
  - Discrete:

$$\sum_{D=0}^{255} H(D) = \#\,\text{of pixels}$$

  - Continuous:

$$\int_{0}^{\infty} H(D)\,dD = \text{area}$$

Properties

- Sum of values between a and b equals the size of all objects in that range:
  - Discrete:

$$\sum_{D=a}^{b} H(D) = \#\text{ of pixels in object(s)}$$

  - Continuous:

$$\int_{a}^{b} H(D)\,dD = \text{area of object(s)}$$

Properties

- Integrated optical density: weight of image (or objects)

$$IOD = \int_{a}^{b} DH(D)\,dD$$

- Mean graylevel: average intensity in image (or objects)

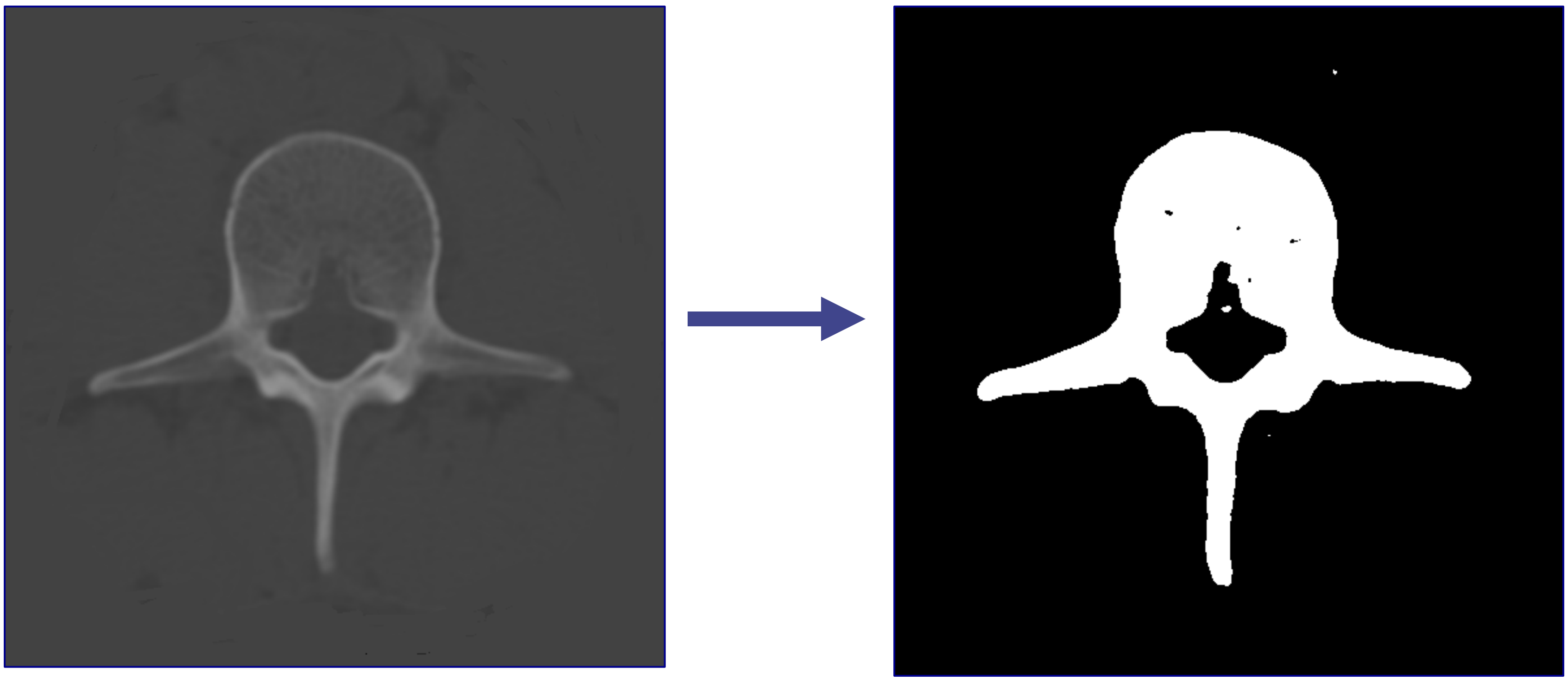
$$MGL = \frac{IOD}{area} = \frac{\int_{a}^{b} DH(D)\,dD}{\int_{a}^{b} H(D)\,dD}$$

Application: Camera Parameters

- Too Bright: lots of pixels at 255 (or max)
- Too Dark: lots of pixels at 0
- Gain Too Low: not enough of the range used

Application: Segmentation

- Can be used to separate bright objects from dark background (or vice versa)



Histograms: Normalizing and Cumulative

- Probability density function: histogram normalized by area

$$p(D) = \frac{H(D)}{A}$$

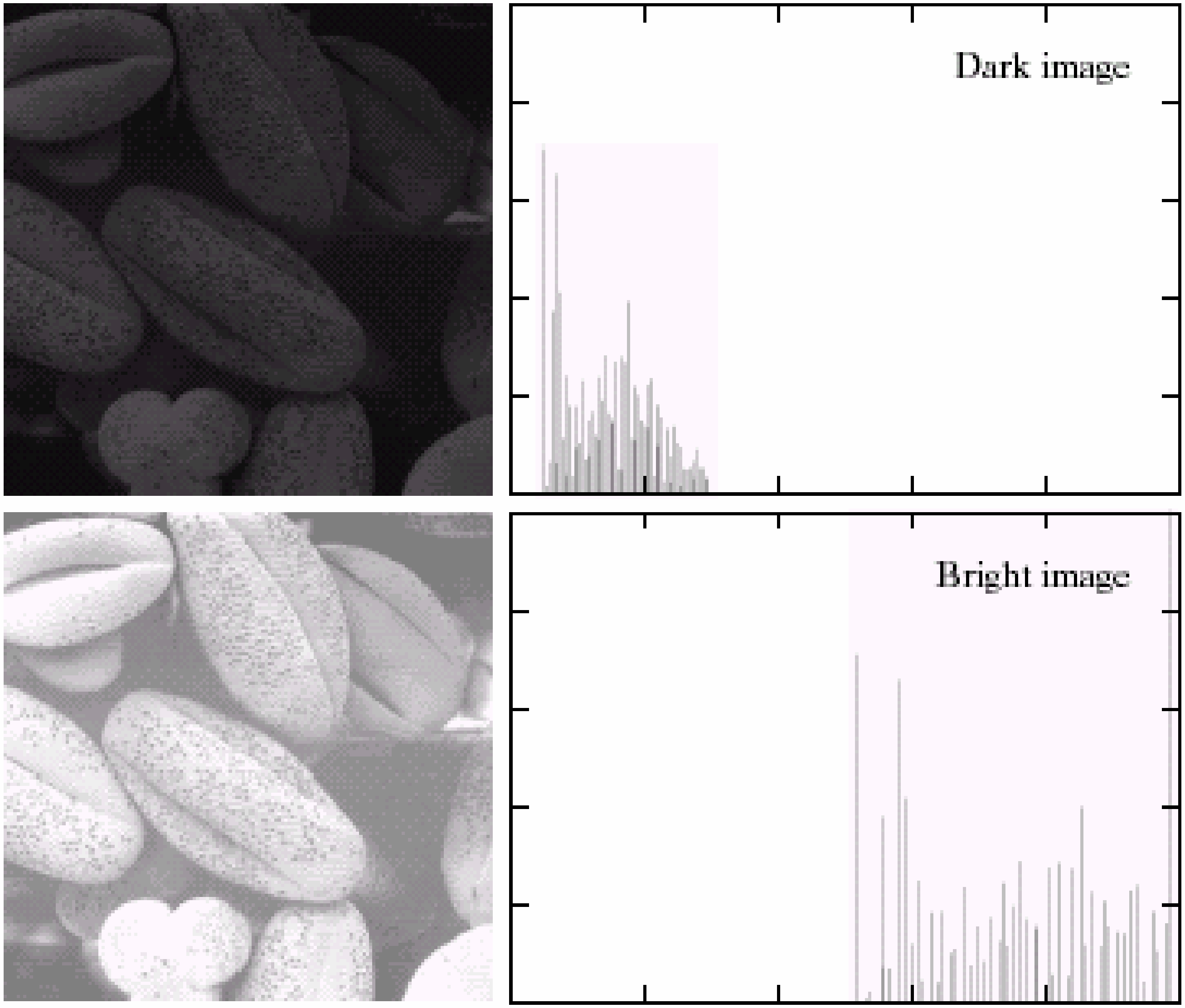
- Cumulative histogram: counts pixels with values up to and including the specified value

$$C(a) = \int_0^a H(D)dD$$

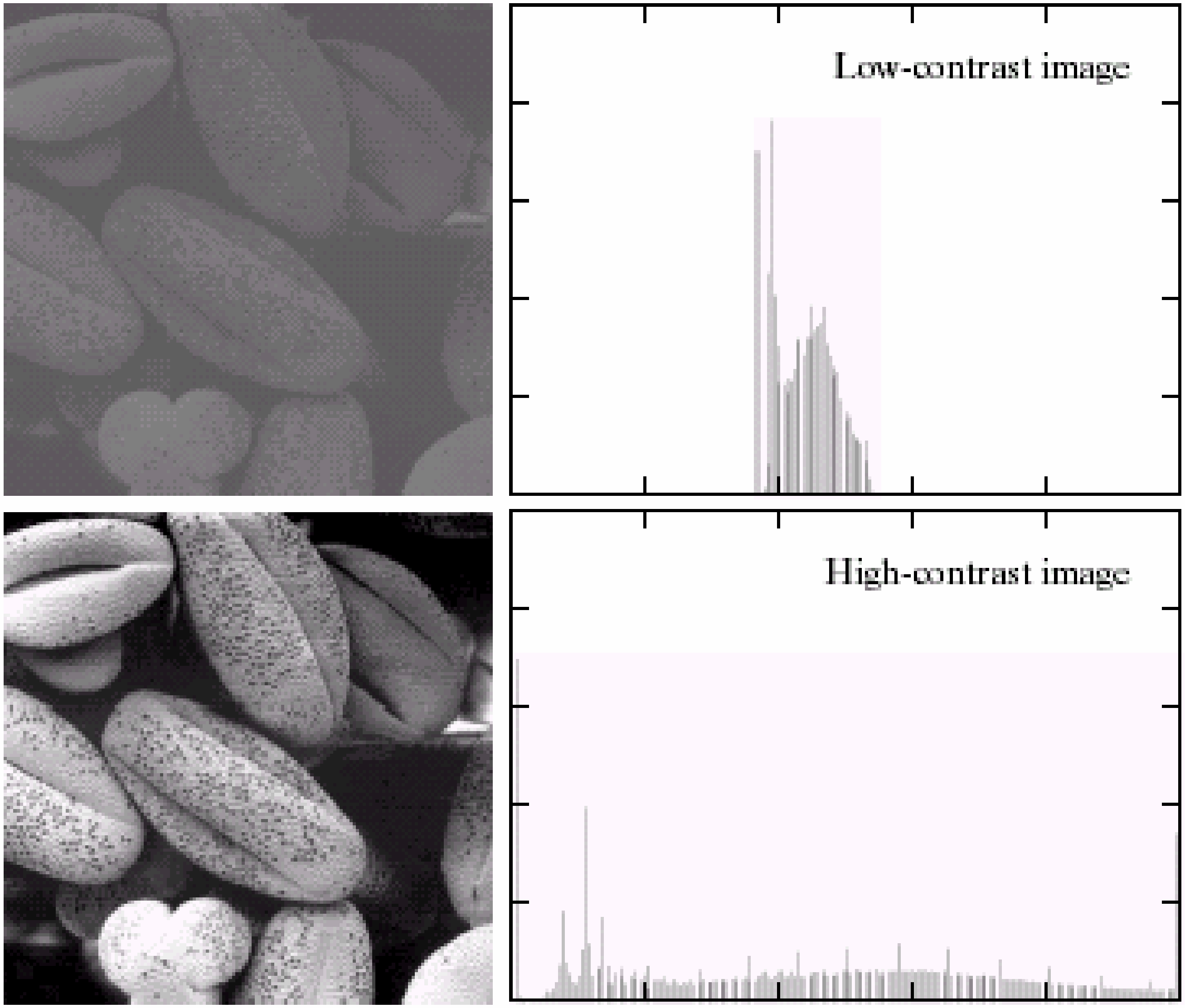
- Cumulative density function: normalized cumulative histogram

$$P(a) = \int_0^a p(D)dD = \frac{C(a)}{A}$$

Histograms, Brightness, and Contrast



Histograms, Brightness, and Contrast



Point Operations on Histograms

Suppose we have a monotonic level operation such that

$$f(a) = \tilde{a}$$

$$f(b) = \tilde{b}$$

Then the histogram H becomes $\widetilde{H}$ such that:

$$\int_{a}^{b} H(g)dg = \int_{\tilde{a}}^{\tilde{b}} \widetilde{H}(g)dg$$

Point Operations on Histograms

Let $b = a + \Delta$ for some very small Δ,

then $f(b) \approx f(a) + f'(a)\Delta$

Thus $$\int_{a}^{a+\Delta} H(g)dg = \int_{\tilde{a}}^{\tilde{a}+f'(a)\Delta} \widetilde{H}(g)dg$$

or approximating the last expression to first order:

$$H(a)\Delta \approx \widetilde{H}(\tilde{a})f'(a)\Delta; \Rightarrow$$

$$\widetilde{H}(\tilde{a}) \approx \frac{H(a)\Delta}{f'(a)\Delta} = \frac{H(a)}{f'(a)}; g \equiv \tilde{a} = f(a) \Rightarrow$$

$$\widetilde{H}(g) = \frac{H(f^{-1}(g))}{f'(f^{-1}(g))}$$

Histogram Equalization

Automatic contrast enhancement:

- Basic Idea: allocate the most output levels to the most frequently occurring inputs
- Look at the histogram of the input signal
- If we allocate output levels proportional to the frequency of occurrence for our input levels, the output histogram should be uniform
- This process is know as histogram equalization

Histogram Equalization

We want a flat (constant) output histogram:

$$\widetilde{H}(g) = \frac{H(f^{-1}(g))}{f'(f^{-1}(g))} = \frac{A_0}{g_{max}}$$

Thus:

$$f'(g) = \frac{g_{max}}{A_0} H(g) \rightarrow f(g) = \frac{g_{max}}{A_0} \int_0^g H(x)dx$$

where

- g is the input gray level
- g_{max} is the maximum input
- A_0 is the image area (area of objects with gray level ≥ 0)
- $f(g)$ is the output gray level

Histogram Equalization

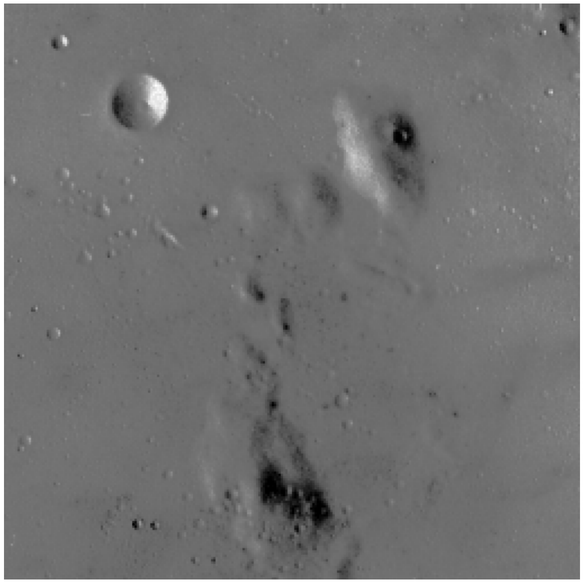
However, the probability density function is the normalized histogram (i.e., $p(g) = H(g) / A_0$):

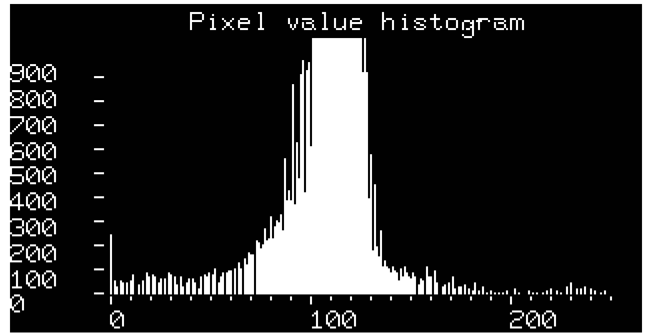
$$f(g) = g_{max} \int_0^g p(x)dx = g_{max} P(g)$$

where

- p is the probability density function (normalized histogram) of the input image
- P is the cumulative probability density function

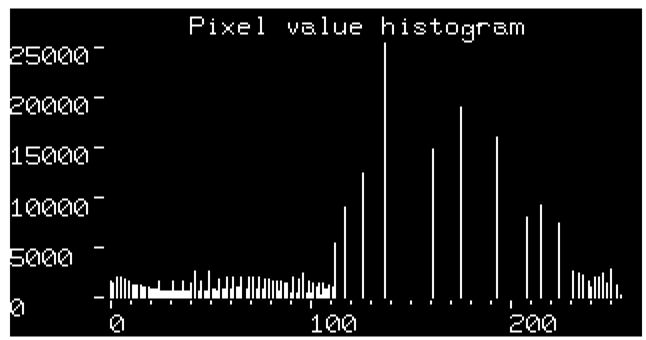
Example of Histogram Equalization





Total number of pixels

Number of intensity density levels



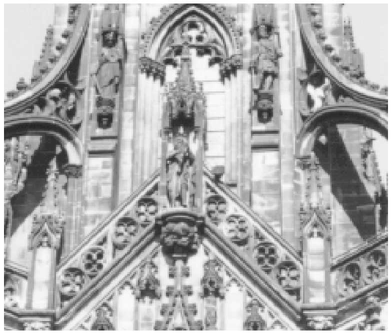
Total number of pixels

Number of intensity density levels

Local Enhancement of HE

Original

General HE enhances
the contrast of sky region,
but building is too dark



Cropped image

HE based on cropped image
enhances the contrast of building
and sky

http://www.cee.hw.ac.uk/hipr/html/histeq.html

Variation: Histogram Matching

- Histogram equalization produces a uniform output histogram
- We can instead make it whatever we want
- Use histogram equalization as an intermediate step
  - First equalize the histogram of the input signal:

$$f_1(g) = g_{max}\, P_1(g)$$

  - Then, equalize the desired output histogram:

$$f_2(g) = g_{max}\, P_2(g)$$

  - Histogram specification (matching) is

$$f(g) = f_2^{-1}(f_1(g)) = P_2^{-1}(P_1(g))$$

Figure and Text Credits

Text and figures for this lecture were adapted in part from the following source, in agreement with the listed copyright statements:

http://web.engr.oregonstate.edu/~enm/cs519

Resources

Textbooks:

Kenneth R. Castleman, Digital Image Processing, Chapter 3, 5

John C. Russ, The Image Processing Handbook, Chapter 3, 4

Resources and Reading Assignment

Textbooks:
Kenneth R. Castleman, Digital Image Processing, Chapter 6, 7, 8
John C. Russ, The Image Processing Handbook, Chapter 5